MATH 201

Multivariable Calculus

Session 2 Fall 2019

Course meeting time and location: Tuesday and Thursday 14:45 – 17:15 in IB 2071

Recitation meeting time and location: Wednesday 14:45 – 16:00 **or** 16:15 – 17:30 in IB 2071

Exam date and location: Tuesday, December 17, 2019, 15:15 – 18:15 in AB 2103 and AB 2107

Academic credit: 4 DKU credits

Course format: Lecture

Instructor's information

Instructor: Prof. Konstantinos Efstathiou, PhD
Office: IB 1020
Office hours: Tuesday and Thursday 13:15 – 14:30 or by appointment
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Personal website: https://efstathiou.gr/
Short bio: Dr. Efstathiou studied physics all the way up to his PhD, focusing on the more mathematical aspects of physics, especially those related to dynamics and geometry. After his PhD, he has worked in the Mathematics departments of the University of Groningen and the Xi'an Jiaotong Liverpool University. His research interests are mostly in the area of dynamical systems where the mathematical tools from multivariable calculus play an essential role in describing and understanding the geometry of the structures that appear in such systems. In this course he is looking forward to convey the essential unity, elegance, and applicability of the concepts in Multivariable Calculus and to demonstrate different techniques for dealing with problems involving many variables.

Recitation instructors:

- Prof. Jian-Guo Liu, PhD (jliu@phy.duke.edu; office: IB 3035).
- Prof. Yuan Gao, PhD (yg86@duke.edu; office: IB 2036).

What is this course about?

In MATH 101 Calculus (also known as Mathematical Foundations 1) you have learned about single variable Calculus—the study of functions that depend on a single variable and whose output is also described by a single variable. This is the basis for large parts of modern Mathematics and applications. However, it does not take long to realize that, despite all its intricacies, single variable calculus is not sufficient for describing many aspects of our world. **Planets and particles** move along trajectories in three-dimensional space (or four-dimensional if you take Relativity into account). **Climate models** describe the state of the atmosphere through quantities such as temperature, atmospheric pressure, and wind velocity—all of them functions of position (longitude, latitude, height) and time. In particular, wind velocity is not a single quantity but a vector—we need to know both how strong the wind is and what is its direction. Neural networks, used in **machine learning**, are trained by minimizing a function that may depend on thousands of variables. In **electromagnetism** the electric and magnetic fields are described by vectors that depend on the position in three-dimensional space and time. To represent volumes, surfaces, and curves in **computer graphics and animation** the concepts from multivariable calculus are indispensable. And from a **purely mathematical point of view**

Multivariable Calculus is a subject that opens the door to almost all of modern mathematics — a beautiful topic with close connections to Geometry.

A very useful mathematical concept for working with many variables is vectors. In this course we will consider vectors and the basic operations between them, and then how to define vector functions that can describe a curve in space and the motion of a particle along such a curve. Then we will consider functions that depend on more than one variables (for simplicity, most of the discussion will be for two or three variables but the concepts can be generalized to any number of variables) and how to differentiate them, leading to the notions of partial derivatives, directional derivatives, and the gradient. We will use these to find minima and maxima of such functions. Then we will look at how to compute integrals for functions that depend on two or three variables (double and triple integrals). We will close with three fundamental results on integration (Green's Theorem, Stokes' Theorem, and the Divergence Theorem) that generalize to more than one dimensions the Fundamental Theorem of Calculus.

What background knowledge do I need before taking this course?

You must have obtained a C- or better in **MATH 101 Calculus (Mathematical Foundations 1)** or have placed out of MATH 101.

What will I learn in this course?

After successfully completing the course you will be able to:

- Interpret real-world situations in terms of related Multivariable Calculus concepts.
- Understand the concept of vectors and its connection to Physics, apply operations on vectors algebraically and geometrically, calculate the dot product and the cross product of vectors.
- Develop analytical and computational skills required for working with lines, curves, planes, and surfaces in space.
- Find limits, partial derivatives, directional derivatives, and the gradient of functions of several variables.
- Understand the definitions of double integrals, triple integrals, line integrals, and surface integrals; recognize and implement appropriate techniques to evaluate them, and apply them to solve problems in mathematics, physical and social life sciences.
- Apply the Fundamental Theorem of Line Integrals, Green's Theorem, Stokes' Theorem, and the Divergence Theorem, to simplify integration problems.

What will I do in this course?

During the **lectures** I will describe the main relevant ideas and in some cases delve into more technical aspects of the topic. The purpose of the lectures is to help you get oriented in Multivariable Calculus by presenting the main ideas, how they are related to each other, and in which directions they can be extended. You are strongly encouraged to ask questions during lectures. During lectures there will also be activities such as "clicker" questions, and reviewing the solutions of homework assignment problems. You are expected to attend class and actively participate in all classroom activities.

During **recitations** an instructor will present the solutions to selected problems related to the theory covered in lectures until that point. The specific problems will be announced on Sakai before the corresponding recitation. The purpose of the recitations is to get familiar with techniques for solving problems in Multivariable Calculus and to better understand the theory through practice.

The assessment of the learning objectives will be performed through a combination of **homework assignments**, **midterm exams**, and the **final exam**.

There will be **6 homework assignments** (handed-out on weeks 1 – 6 with deadline for handing them in one week later). The purpose of the homework assignments is to make you actively think about the covered material and to assess your progress, providing also signs on how well the learning objectives are attained, and you will be provided with detailed feedback on selected problems that will be graded. You can work on the homework assignments alone or with your fellow students but you must hand-in individually written assignments.

There will be **two midterm exams**. Midterm exam 1 will assess learning objectives related to Chapters 12 and 13 in the textbook, and midterm exam 2 related to Chapters 14 and 15. The **final exam** will assess learning objectives for the whole course, including Chapter 16 in the textbook.

How can I prepare for the class sessions to be successful?

To prepare for the **course**:

- Review MATH 101 Mathematical Foundations 1.

To prepare for the **lectures**:

- Make sure that you have a thorough understanding of the material covered in previous sessions and prepare a list of questions concerning aspects of the material that is still unclear. You can ask these questions during the lecture or recitation, or during office hours.
- Read through the corresponding upcoming sections in the textbook (the lecture schedule is given later in this syllabus) to get an idea of what will be discussed.
- Study auxiliary resources (for example, videos, or discussion chapters from the textbook) that will be communicated to you through Sakai before the lectures.

To prepare for the **recitations**:

- Review the theory of the two previous lectures so that you are more familiar with the concepts that will be covered in the solved problems.
- Make a first attempt to solve the selected problems announced on Sakai before the recitation. Make a list of questions concerning difficult points.

To succeed, you should be prepared to devote several hours to this course on a daily basis. You are strongly encouraged to use the tutoring resources of ARC and to contact me in a timely manner for additional help as needed.

What required texts, materials, and equipment will I need?

The main (required) textbook for this course is:

- James Stewart, **Calculus: Early Transcendentals**, 8th Edition, ISBN-13: 978-1285741550.

  Stewart's book is very detailed and contains several applications of the theory. Moreover, it contains a huge amount of questions that you can use to practice. However, it misses some topics that I find essential for a proper understanding of Multivariable Calculus (such as Taylor series, the Hessian, derivatives as matrices, the chain rule as matrix multiplication, the Implicit and Inverse Function Theorems, and the Morse Lemma). I will provide extra notes to cover the missing material.

What optional texts or resources might be helpful?

- Susan Jane Colley, **Vector Calculus**, Pearson, 4th edition, 2012.

  Compared to Stewart this is a more mathematically oriented book and it puts somewhat less emphasis on applications. Colley's book covers most of the missing material that I mention above (except for the Morse Lemma) and can provide an extra source of information for these topics.

- Michael Spivak, **Calculus on Manifolds**, Addison-Wesley, 1965.

  This is a nice little book to go through *after* you have completed the MATH 201 course if you are really interested to dive deeper into the Mathematics. It presents most of the concepts and results that we will discuss in MATH 201 from a more abstract and unified point of view. However, it lacks in motivation and presents no applications of the theory. The book's subtitle, "A Modern Approach to Classical Theorems of Advanced Calculus", refers to the Green's, Stokes', and Divergence Theorems, which are all special versions of a more general mathematical result stated by Élie Cartan in 1945 and known, rather confusingly, as the (Generalized) Stokes' Theorem.

How will my grade be determined?

The grade will be determined through the following graded assessments:

Homework assignments (100 points = 5 x 20 points). There will be 6 compulsory homework assignments and the 5 best will count toward the grade. 10 points are for completeness (answering every problem in the homework assignment) and 10 points are for correctness (correct replies in a selected number of problems).
Midterm exam 1 (50 points).
Midterm exam 2 (50 points).
Final exam (100 points).

Grading Scale

Grade	Percentage (p)
A+	97% ≤ p ≤ 100%
A	93% ≤ p < 97%
A-	90% ≤ p < 93%
B+	87% ≤ p < 90%
B	83% ≤ p < 87%
B-	80% ≤ p < 83%

C+	77% ≤ p < 80%
C	73% ≤ p < 77%
C-	70% ≤ p < 73%
D+	67% ≤ p < 70%
D	63% ≤ p < 67%
D-	60% ≤ p < 63%
F	p < 60%

What are the course policies?

Academic Integrity:
As a student, you should abide by the academic honesty standard of the Duke Kunshan University. Its Community Standard states: "Duke Kunshan University is a community comprised of individuals from diverse cultures and backgrounds. We are dedicated to scholarship, leadership, and service and to the principles of honesty, fairness, respect, and accountability. Members of this community commit to reflecting upon and upholding these principles in all academic and non-academic endeavors, and to protecting and promoting a culture of integrity and trust."

Academic Policy & Procedures:
You are responsible for knowing and adhering to academic policy and procedures as published in University Bulletin and Student Handbook. Please note, an incident of behavioral infraction or academic dishonesty (cheating on a test, plagiarizing, etc.) will result in immediate action from me, in consultation with university administration (e.g., Dean of Undergraduate Studies, Student Conduct, Academic Advising). Please visit the Undergraduate Studies website for additional guidance related to academic policy and procedures.

Academic Disruptive Behavior and Community Standard:
Please avoid all forms of disruptive behavior, including but not limited to: verbal or physical threats, repeated obscenities, unreasonable interference with class discussion, making/receiving personal phone calls, text messages or pages during class, excessive tardiness, leaving and entering class frequently without notice of illness or other extenuating circumstances, and persisting in disruptive personal conversations with other class members. Please turn off phones, pagers, etc. during class unless instructed otherwise. Laptop computers may be used only to take notes and to consult the textbook. If you choose not to adhere to these standards, I will take action in consultation with university administration (e.g., Dean of Undergraduate Studies, Student Conduct, Academic Advising).

Academic Accommodations:
If you need to request accommodation for a disability, you need a signed accommodation plan from Campus Health Services, and you need to provide a copy of that plan to me. Visit the Office of Student Affairs website for additional information and instruction related to accommodations.

What campus resources can help me during this course?

Academic Advising and Student Support

Please consult with me about appropriate course preparation and readiness strategies, as needed. Consult your academic advisors on course performance (i.e., poor grades) and academic decisions (e.g., course changes, incompletes, withdrawals) to ensure you stay on track with degree and graduation requirements. In addition to advisors, staff in the Academic Resource Center can provide recommendations on academic success strategies (e.g., tutoring, coaching, student learning preferences). Please visit the Office of Undergraduate Advising website for additional information related to academic advising and student support services.

Writing and Language Studio

If you want additional help with academic writing – and more generally with language learning – you are welcome to go to the Writing and Language Studio (WLS), located in the Conference Center.

What is the expected course schedule?

The numbers in the brackets under "class topic / unit name" below refer to sections in the course textbook. The schedule may change as the course progresses. Such changes will be announced on Sakai and the syllabus will be revised. Starred items correspond to extra material that will be covered if there is sufficient time.

Date	Class topic / unit name	Planned in-class activities	Assignments due
Week 1			
Lecture 1.1	Three-Dimensional Coordinate Systems (12.1) Vectors (12.2) The Dot Product (12.3) The Cross Product (12.4)	Lecture	
Recitation 1		Solve problems	
Lecture 1.2	Equations of Lines and Planes (12.5) Cylinders and Quadric Surfaces (12.6) Vector Functions and Space Curves (13.1)	Lecture	
Week 2			
Lecture 2.1	Derivatives and Integrals of Vector Functions (13.2) Arc Length and Curvature (13.3) Motion in Space: Velocity and Acceleration (13.4)	Lecture	
Recitation 2		Solve problems	**Homework 1**
Lecture 2.2	Functions of Several Variables (14.1) Limits and Continuity (14.2) Partial Derivatives (14.3)	Lecture	
Week 3			

Lecture 3.1	**Midterm Exam 1** Tangent Planes and Linear Approximations (14.4)	**In-class exam** Lecture	
Recitation 3		Solve problems	**Homework 2**
Lecture 3.2	The Chain Rule (14.5) Implicit Function Theorem (*) Directional Derivatives and the Gradient Vector (14.6)	Lecture	
Week 4			
Lecture 4.1	Maximum and Minimum Values (14.7) Hessian and Morse Lemma (*) Lagrange multipliers (14.8)	Lecture	
Recitation 4		Solve problems	**Homework 3**
Lecture 4.2	Double integrals over rectangles (15.1) Double Integrals over General Regions (15.2) Double Integrals in Polar Coordinates (15.3)	Lecture	
Week 5			
Lecture 5.1	Triple Integrals (15.6) Triple Integrals in Cylindrical Coordinates (15.7) Triple Integrals in Spherical Coordinates (15.8)	Lecture	
Recitation 5		Solve problems	**Homework 4**
Lecture 5.2	Vector Fields (16.1) Line Integrals (16.2) The Fundamental Theorem for Line Integrals (16.3)	Lecture	
Week 6			
Lecture 6.1	**Midterm Exam 2**	**In-class exam**	
Recitation 6		Solve problems	**Homework 5**
Lecture 6.2	Green’s Theorem (16.4) Curl and Divergence (16.5) Parametric Surfaces and Their Areas – Part 1 (16.6)	Lecture	
Week 7			
Lecture 7.1	Parametric Surfaces and Their Areas – Part 2 (16.6) Surface Integrals (16.7) Stokes’ Theorem (16.8)	Lecture	
Recitation 7		Solve problems	**Homework 6**
Lecture 7.2	The Divergence Theorem (16.9) Review	Lecture	