MATH 302

Numerical Analysis

Fall 2024, Session 1

昆山杜克大学
DUKE KUNSHAN UNIVERSITY

Class meetings: Tuesday and Thursday 12:00 – 14:30 in LIB 1119.
Programming labs: Thursday 18:00 – 19:15 in LIB 1119.
Academic credit: 4 DKU credits.
Course format: Lectures, Programming Lab.

Course Instructor

Instructor: Konstantinos Efstathiou, PhD.
Office: WDR 3107.
Office hours: Monday 13:00 - 14:30 and Thursday 15:00 - 16:30 or by appointment.
Personal Zoom room: https://duke.zoom.us/my/k.efstathiou (for online "office hour" meetings).
E-mail address: k.efstathiou@dukekunshan.edu.cn.
Personal website: https://www.efstathiou.gr/.

Course TA: Shuai Dong; E-mail: shuai.dong@dukekunshan.edu.cn.

Short bio: Dr. Efstathiou studied physics all the way up to his PhD, focusing on the more mathematical aspects of physics, especially those related to dynamics and geometry. He has worked in the Mathematics departments of Xi'an Jiaotong Liverpool University and the University of Groningen, before joining DKU. His research concerns the geometric aspects of Hamiltonian dynamical systems and the synchronization and other collective behavior of coupled oscillator systems. His experience with Numerical Analysis did not start with a course. It started when as a first year undergraduate student he read about the Lorenz attractor and tried to plot it. As he was getting more interested in the study of dynamical systems he needed more methods for the numerical study of different problems and he developed a deep interest on how these methods work and what are their main characteristics. He hopes to convey some of the most interesting aspects of the topic in this course.

What is this course about?

Predicting the weather; deciding the optimal value of a financial asset; sending spacecrafts to asteroids and to the edge of the solar system. These are only a few of the tasks that are achieved through a combination of mathematical modeling and numerical simulations. In this course we will focus on the theoretical foundations and practical aspects of such numerical simulations.

First, we will discuss the most fundamental and widely used algorithms for numerical simulations. This being a mathematically focused course, we will go beyond a "numerical recipes" approach. We will discuss the main practical and theoretical considerations for designing numerical algorithms (stability, accuracy, computational cost) and delve deep into the details of each algorithm.

Second, we will not shy away from the practical aspects of implementing numerical algorithms in an efficient manner. For the implementation of the algorithms we will be working with the Julia programming language. This is a modern dynamic language for numerical computations which combines expressiveness and speed.

In terms of its placement within the curriculum, MATH 302 requires a solid foundation in MATH 201 Multivariable Calculus and MATH 202 Linear Algebra. Moreover, it requires basic programming skills which are covered in courses such as COMPSCI 101 Introduction to Computer Science or COMPSCI 201 Introduction to Programming and Data Structures or STATS 102 Introduction to Data Science. The knowledge from MATH 302 can be useful in several other courses in the mathematics curriculum and beyond. For example, MATH 303 ODE and Dynamical Systems and MATH 406 Mathematical Modeling use many of the numerical methods that we will discuss here. Moreover, a strong computational background, founded on deep understanding, can also be valuable for Signature Work.

Why Julia?

Julia is a relatively new programming language which combines the ease of Python, with the speed of C. As a language designed specifically for scientific computing, Julia offers facilities that are missing in Python and many other languages, while it can be much faster than standard Python. Even though Julia's syntax has similarities with Python, you should also be aware of a few differences. From a programming language design point of view, Julia adopts certain interesting ideas such as multiple dispatch and macros; in this course, however, we will use only structured programming concepts.

What background knowledge do I need before taking this course?

MATH 302 has as prerequisites:

- ☑ MATH 201 Multivariable Calculus.
- ☑ MATH 202 Linear Algebra.
- ☑ COMPSCI 101 Introduction to Computer Science or COMPSCI 201 Introduction to Programming and Data Structures or STATS 102 Introduction to Data Science.

Moreover, it will be useful (but not strictly required) to have some prior familiarity with:

- ☑ Criteria for convergence of sequences of real numbers (as discussed, for example, in MATH 203 or MATH 205 or MATH 308).
- ☑ LaTeX (for the applied project written report).

What will I learn in this course?

After successfully completing the course, you will be able to:

1. Recall the fundamental definitions, results, and types of algorithms from numerical analysis.
2. Reproduce the proofs of the main theoretical results and demonstrate understanding.
3. Implement an algorithm in a specific programming language so that it can efficiently run on a computer.
4. Analyze the efficiency and complexity of an algorithm.
5. Evaluate different algorithms for use in a specific problem in terms of their trade-offs.
6. Apply the learned algorithms and techniques to solve problems with an interdisciplinary character.

7. Integrate knowledge from the course and literature research into a project level work on a specific numerical analysis topic.

What will I do in this course?

Each week we will have **2 class meetings** and **1 programming lab session**. In the class meetings we will review the main theoretical results and discuss examples and the implementation of the relevant algorithms. In the **programming lab sessions** you will work in pairs to implement or evaluate algorithms using the Julia programming language. The aim of these lab sessions is for you to better understand several aspects of the algorithms and techniques we learn in the course, to implement them in Julia and explore their properties, and to improve or hone your Julia programming skills. After each lab you will need to write a short **lab report** on the work you did during the lab which will be graded as Pass / Fail.

Moreover, we will have graded **in-class quizzes** in almost every class meeting. The purpose of these quizzes will be to assess your recall of basic facts and definitions and fundamental understanding through multiple choice questions. In-class quizzes will be closed book, closed laptop, but you may use a handheld calculator with support for scientific functions. Moreover, in each session there will be several ungraded polls.

Each week there will be a **homework assignment**. The purpose of the homework assignments is to work on Numerical Analysis problems without tight time limitations and possibly in collaboration with your fellow students. Note however that you must submit your own, independently written, answers. Each answer in the homework assignments will be graded as Pass / Fail based on whether the provided answer is reasonable and demonstrates a serious engagement with the question. Complete answers to all homework problems will be provided after each homework assignment deadline so you can compare with your answers.

Another part of the course will be a short **project**. The purpose of the project is to study deeper one particular Numerical Analysis topic. The project deliverable is a **written report** on the chosen topic. A list of possible topics from which to choose and specific requirements for the report will be given in week 2.

Finally, we will have a **midterm exam** and a **final exam**. The exams will ask questions about theoretical aspects of the algorithms and their implementation, possibly including programming questions. They aim to assess your own ability to solve problems and to synthesize knowledge in a time-constrained setting and they build upon the in-class discussions, quizzes, and homework assignments.

Finally, you should make ample use of **office hours** and other opportunities to interact and ask questions (e.g., on Canvas discussions). If the given office hours do not work for you, send me an e-mail to arrange an appointment. My role is to guide and support your learning — do not hesitate to come to me with your questions and comments.

How can I prepare for the class sessions to be successful?

To prepare for class sessions:

- ☑ At the beginning of the course I will hand out a reading guide about the material that you should focus on from each covered section; a list of the sections that will be covered is at the end of this syllabus. **I expect that you come to the class meetings having gone through the reading guide,**

having read the corresponding sections from the textbook, and having watched any required videos.

- ☑ **Bring your laptop and a handheld calculator to class**. The calculator will be useful for the in-class quizzes.
- ☑ Make sure that you have a thorough understanding of the material covered in previous class meetings and prepare a list of questions concerning aspects of the material that is still unclear. You can ask these questions during the following class meeting or during office hours.
- ☑ Read carefully through the corresponding upcoming sections in the textbook (the lecture schedule is given later in this syllabus) and prepare questions to ask during the lectures. I will review this material in every lecture, focusing on the main ideas, but I will not go through every point in detail. In case some discussion in the textbook is unclear, you can raise your questions in class (preferable) or during office hours.
- ☑ Study auxiliary resources (for example, videos) that will be communicated to you through Canvas before the lectures.

To prepare for the project:

- ☑ Make sure that you know how to produce a written report using LaTeX; I suggest using a (free) Overleaf account instead of locally installing LaTeX on your computer.

To prepare for the midterm and final exams:

- ☑ Review the solved problems from homework assignments and the problems solved in class. Make sure that you have mastered the problems. You can check this by being able to solve the problems without checking at all the given solutions.

What required texts, materials, and equipment will I need?

The following textbook is required:

- **Numerical Analysis** by Richard L. Burden, J. Douglas Faires, and Annette M. Burden. 10th edition, 2016. Cengage.

It is also required to:

- Install Visual Studio Code, Julia (version 1.10 or later), and the Visual Studio Code support for Jupyter notebooks on your laptop.
- Get a (free) Overleaf account for the class project.
- Bring your laptop to every class meeting. Handheld calculators may also be useful for the in-class quizzes.

Moreover, it is suggested to:

- Install the Gradescope mobile app.

What optional texts or resources might be helpful?

- **Fundamentals of Numerical Computation: Julia Edition** by Tobin A. Driscoll and Richard J. Braun. 2022. Society for Industrial and Applied Mathematics.
- **Elementary Numerical Analysis** by Kendall Atkinson and Weimin Han. 3rd edition, 2003. Wiley.

Some online resources for learning Julia include:

- The YouTube video tutorial Intro to Julia (version 1.0) by Jane Herriman is very comprehensive at almost 2.5 hours long. The most relevant part of the tutorial starts at 20:34 and ends at 1:29:25; it covers the basics of structured programming with Julia. The accompanying Jupyter notebooks can be found on GitHub. The website JuliaBox mentioned in the tutorial has been replaced by JuliaHub which can be used instead of installing Julia and Visual Studio Code locally. **I strongly suggest to go through the tutorial before classes begin.**
- The Julia Programming Language website has several learning resources, including a getting started guide.
- The website Calculus with Julia uses Julia to teach Calculus, but it can also be used to learn Julia through Calculus.

Other resources

- Blog post Exposing Floating Point about how the IEEE 754 floating point representation of numbers works and accompanying website float.exposed.

How will my grade be determined?

The course grade will be determined through the following graded assessments:

- Final exam: 40%
- Midterm exam: 24%
- Project report: 10%
- Homework assignments: 10%
- Short in-class quizzes: 8%
- Lab reports: 8%

Grading scale

Grade	Percentage (p)	Grade	Percentage (p)	Grade	Percentage (p)	Grade	Percentage (p)
A+	$97\% \le p \le 100\%$	B+	$87\% \le p < 90\%$	C+	$77\% \le p < 80\%$	D+	$67\% \le p < 70\%$
A	$93\% \le p < 97\%$	B	$83\% \le p < 87\%$	C	$73\% \le p < 77\%$	D	$63\% \le p < 67\%$
A-	$90\% \le p < 93\%$	B-	$80\% \le p < 83\%$	C-	$70\% \le p < 73\%$	D-	$60\% \le p < 63\%$
						F	$p < 60\%$

What are the course policies?

Communications

If you have a question or discussion topic that you think could be relevant to the whole class then I would like to encourage you to ask it in Canvas Discussions so that everyone can take part. I would also like to encourage you to reply to questions if you know the answer. For questions that are only relevant to you I prefer e-mail. Note that I usually do not answer e-mails in the evenings and the weekends unless there is something urgent.

Assignment deadlines

All homework assignments and the final version of the applied project written report should be handed-in by the announced deadline; late submissions will not be graded unless an extension has been requested at least 24 hours before the deadline and a convincing explanation has been provided. An extension will never be longer than 24 hours.

It is especially important to submit homework assignments within the given deadline. The reason is that after I have received all assignments I can then start grading and providing feedback and I can also release the solutions to the assignments so that you can compare with your solutions.

Discussion Guidelines

Civility is an essential ingredient for academic discourse. All communications for this course should be conducted constructively, civilly, and respectfully. Differences in beliefs, opinions, and approaches are to be expected. Please bring any communications you believe to be in violation of this class policy to the attention of your instructor. Active interaction with peers and your instructor is essential to success in this course, paying particular attention to the following:

- ☑ Be respectful of others and their opinions, valuing diversity in backgrounds, abilities, and experiences.
- ☑ Challenging the ideas held by others is an integral aspect of critical thinking and the academic process. Please word your responses carefully, and recognize that others are expected to challenge your ideas. A positive atmosphere of healthy debate is encouraged.
- ☑ Read your online discussion posts carefully before submitting them.

Academic Integrity

As a student, you should abide by the academic honesty standard of the Duke Kunshan University. Its Community Standard states: “Duke Kunshan University is a community comprised of individuals from diverse cultures and backgrounds. We are dedicated to scholarship, leadership, and service and to the principles of honesty, fairness, respect, and accountability. Members of this community commit to reflecting upon and upholding these principles in all academic and non-academic endeavors, and to protecting and promoting a culture of integrity and trust.”

Academic Policy & Procedures

You are responsible for knowing and adhering to academic policy and procedures as published in University Bulletin and Student Handbook. Please note, an incident of behavioral infraction or academic dishonesty (cheating on a test, plagiarizing, etc.) will result in immediate action from me, in consultation with university administration (e.g., Dean of Undergraduate Studies, Student Conduct, Academic Advising). Please visit the Undergraduate Studies website for additional guidance related to academic policy and procedures.

Academic Disruptive Behavior and Community Standard

Please avoid all forms of disruptive behavior, including but not limited to: verbal or physical threats, repeated obscenities, unreasonable interference with class discussion, making/receiving personal phone calls, text messages or pages during class, excessive tardiness, leaving and entering class frequently without notice of illness or other extenuating circumstances, and persisting in disruptive personal conversations with other class members. Please turn off phones, pagers, etc. during class unless instructed otherwise. Laptop computers may be used only to take notes. If you choose not to adhere to these standards, I will take action in consultation with university administration (e.g., Dean of Undergraduate Studies, Student Conduct, Academic Advising).

Academic Accommodations

If you need to request accommodation for a disability, you need a signed accommodation plan from Campus Health Services, and you need to provide a copy of that plan to me. Visit the Office of Student Affairs website for additional information and instruction related to accommodations.

What campus resources can help me during this course?

Academic Advising and Student Support

Please consult with me about appropriate course preparation and readiness strategies, as needed. Consult your academic advisors on course performance (i.e., poor grades) and academic decisions (e.g., course changes, incompletes, withdrawals) to ensure you stay on track with degree and graduation requirements. In addition to advisors, staff in the Academic Resource Center can provide recommendations on academic success strategies (e.g., tutoring, coaching, student learning preferences). All ARC services will continue to be provided online. Please visit the Office of Undergraduate Advising website for additional information related to academic advising and student support services.

Writing and Language Studio

For additional help with academic writing—and more generally with language learning—you are welcome to make an appointment with the Writing and Language Studio (WLS). You can register for an account, make an appointment, and learn more about WLS services, policies, and events on the WLS website. You can also find writing and language learning resources on the Writing & Language Studio Sakai site.

IT Support

If you are experiencing technical difficulties, please contact IT:

- China-based faculty / staff / students 400-816-7100, (+86) 0512-3665-7100
- US-based faculty / staff / students (+1) 919-660-1810
- International-based faculty / staff / students can use either telephone option (recommend using tools like Skype calling)
- Live Chat: https://oit.duke.edu/help
- Email: service-desk@dukekunshan.edu.cn

What is the expected course schedule?

Numbers of the form §N.M below refer to sections in the course textbook.

Week 1

Class meeting 1-1

Topics	Course overview. Review of Calculus (§1.1). Round-off errors and computer arithmetic (§1.2).
Pre-class work	Review the syllabus. Review sections §1.1, §1.2. Install Julia, Visual Studio Code, and the Visual Studio Code support for Jupyter notebooks.
Planned in-class activities	Introduction, course overview, and syllabus review. Short review of some Calculus topics, especially Taylor series and mean value theorems. Discussion of the IEEE 754 floating point representation, and round-off errors in numerical computations due to the finite precision of IEEE 754.

Class meeting 1-2

Topics	Algorithms and convergence (§1.3). Bisection method (§2.1). Fixed point iteration (§2.2).
Pre-class work	Review sections §1.3, §2.1, §2.2.
Planned in-class activities	Short quiz. Discussion of §1.3 including algorithms, pseudo-code, and how we characterize the convergence properties of algorithms. Discussion of methods for approximating the solutions to equations of the form $f(x) = 0$, starting with the bisection method from §2.1 and fixed point iteration from §2.2.

Lab session 1

Topics	Julia language fundamentals.
Pre-class work	Go through the tutorial Intro to Julia (version 1.0) by Jane Herriman. This video tutorial is about 2.5 hours long and it will take more time to follow along in a Jupyter notebook so I advise you to start going through the tutorial before the start of classes. You should make sure before the lab that you can run Jupyter notebooks in Visual Studio Code.
Planned in-class activities	In this lab you will be given simple programming assignments that you will then implement in Julia.

Week 2

Deadline Homework 1 (Monday).

Class meeting 2-1

Topics	Fixed point iteration (§2.2). Newton's method and its extensions (§2.3). Error analysis for iterative methods (§2.4).
Pre-class work	Review sections §2.2, §2.3, §2.4.
Planned in-class activities	Short quiz. Discussion of methods for solving equations of the form $f(x) = 0$ focusing on Newton's method, the secant method, and the convergence properties of fixed point methods.

Class meeting 2-2

Topics	Fixed points for functions of several variables (§10.1). Newton's method (§10.2). Steepest descent techniques (§10.4).
Pre-class work	Review sections §10.1, §10.2, §10.4.
Planned in-class activities	Short quiz. Discussion of how to solve systems of nonlinear equations using a generalization of Newton's method (§10.2) and the steepest descent method (§10.4).

Lab session 2

Topics	Comparison of root finding methods.
Pre-class work	Review the provided implementations of root finding methods. Install the Julia package `Plots` and find documentation about plotting with it. Find documentation about using arbitrary precision floating point numbers (`BigFloat`) in Julia.
Planned in-class activities	In this lab you will work to compare different root finding methods in terms of speed and accuracy and present the results in a visual way.

Week 3

Deadline Homework 2 (Monday).

Class meeting 3-1

Topics	Interpolation and the Lagrange polynomial (§3.1). Divided differences (§3.3). Hermite interpolation (§3.4).
Pre-class work	Review sections §3.1, §3.3, §3.4.
Planned in-class activities	Short quiz. Discussion of polynomial interpolation methods.

Class meeting 3-2

Topics Cubic spline interpolation (§3.5).
Numerical differentiation (§4.1).

Pre-class work Review sections §3.5, §4.1.

Planned in-class activities Short quiz. Discussion of cubic spline interpolation methods. Discussion of numerical differentiation.

Lab session 3

Topics Exploration of the properties of interpolating polynomials.

Pre-class work Review the provided implementations of interpolation methods.

Planned in-class activities In this lab you will work to explore the effect of the choice of nodes in the construction of an interpolating polynomial.

Week 4

Deadline Homework 3 (Monday noon).

Class meeting 4-1

Midterm Exam In-class exam, covers material from weeks 1-3.

Topics In-class midterm exam and SGIF.

Pre-class work Prepare for the exam.

Planned in-class activities In this class meeting we will have an in-class exam that will cover the material from weeks 1-3. Then, a **SGIF** will be organized by the CTL for midterm course feedback.

Class meeting 4-2

Topics Richardson's extrapolation (§4.2).
Elements of numerical integration algorithms (§4.3).
Composite numerical integration (§4.4).
Romberg integration (§4.5).

Pre-class work Review sections §4.2, §4.3, §4.4, §4.5.

Planned in-class activities *There will not be a short quiz in this class meeting*. Discussion of Richardon's extrapolation, and various numerical integration schemes.

Lab session 4

Topics Exploration of round-off error in numerical differentiation.

Pre-class work Review how to plot data and how to get new arrays in Julia by applying a function to an existing array.

Planned in-class activities In this lab you will compare numerical differentiation methods and explore the role of round-off errors and you will visualize your results.

Week 5

Deadline Homework 4 (Monday).

Class meeting 5-1

Topics The Elementary Theory of Initial-Value Problems (§5.1)
Euler's method (§5.2).
Higher order Taylor methods (§5.3).

Pre-class work Review sections §5.1, §5.2, §5.3

Planned in-class activities Short quiz. Discussion of some methods for the numerical solution of ordinary differential equations.

Class meeting 5-2

Topics Runge-Kutta methods (§5.4).
Error control and the Runge-Kutta-Fehlberg method (§5.5).
Higher-order equations and systems of differential equations (§5.9).
Finite-difference methods for linear problems (§11.3).

Pre-class work Review sections §5.4, §5.5, §5.9, §11.3.

Planned in-class activities Short quiz. Discussion of Runge-Kutta methods for initial value problems and systems of differential equations and finite-difference methods for boundary value problems.

Lab session 5

Topics Exploration of the properties of ODE integration methods.

Pre-class work Review the provided implementations of the Euler, midpoint, and Runge-Kutta methods.

Planned in-class activities In this lab you will work to study the local truncation error of ODE integration methods.

Week 6

Deadline Homework 5 (Monday).

Class meeting 6-1

Topics	Linear systems of equations (§6.1). Pivoting strategies (§6.2). Linear algebra and matrix inversion (§6.3). The determinant of a matrix (§6.4). Matrix factorization (§6.5).
Pre-class work	Review sections §6.1 – §6.5. Most of the material about eigenvalues and eigenvectors in §6.3 and §6.4 is already known from Linear Algebra and will not be repeated in class.
Planned in-class activities	Short quiz. Discussion of direct methods for solving linear systems, focusing on different pivoting strategies and when to use each one of them, and LU factorization.

Class meeting 6-2

Topics	Special types of matrices (§6.6). Norms of vectors and matrices (§7.1). Eigenvalues and eigenvectors (§7.2).
Pre-class work	Review sections §6.6, §7.1, §7.2. Most of the material about eigenvalues and eigenvectors in §7.2 is already known from Linear Algebra and will not be repeated in class.
Planned in-class activities	Short quiz. Discussion of direct methods for special types of matrices. Discussion of vector and matrix norms and their relation to the spectral radius and convergence, as preparation for the discussion of iterative methods in the following class meeting.

Lab session 6

Topics	Implementation of a factorization algorithm.
Pre-class work	Review the provided implementations of LU factorization.
Planned in-class activities	In this lab you will implement Cholesky factorization in Julia.

National Day Holiday

Week 7

Deadline Homework 6 (Monday).

Class meeting 7-1

Topics	The Jacobi and Gauss-Seidel iterative techniques (§7.3). Relaxation techniques for solving linear systems (§7.4). Error bounds and iterative refinement (§7.5).
Pre-class work	Review sections §7.3, §7.4, §7.5.
Planned in-class activities	Short quiz. Discussion of iterative methods for solving linear systems and what are the main ideas behind such methods.

Class meeting 7-2

Topics	Discrete least squares approximation (§8.1). Orthogonal Polynomials and Least Squares Approximation (§8.2). Chebyshev Polynomials and Economization of Power Series (§8.3).
Pre-class work	Review sections §8.1, §8.2, §8.3.
Planned in-class activities	Short quiz. Discussion of polynomial approximation of data and functions.

Lab session 7

Topics	Comparison of direct and iterative methods.
Pre-class work	Review the provided implementations of direct and iterative methods for solving linear systems. Find out how to produce random strictly diagonally dominant matrices of arbitrary size in Julia.
Planned in-class activities	In this lab you will compare a direct and an iterative method in terms of computational cost and how this scales with the size of the system to solve.

Deadline The **project report** is due **Friday 11 October, 23:59**.

Exam Week

Deadline Homework 7 (Monday).

Final Exam Thursday, October 17, 2024, 15:30 – 18:30, IB 1046.