## MATH 308

# Real Analysis

### Fall 2023, Session 1

昆山杜克大学
DUKE KUNSHAN UNIVERSITY

**Class meeting time:** Monday and Wednesday 12:00 – 14:30 in WDR 1007
**Academic credit:** 4 DKU credits
**Course format:** Lectures

## Course Instructor

**Instructor:** Konstantinos Efstathiou, PhD
**Office:** WDR 3107
**Office hours:** Tuesday 10:00 - 11:00 and Thursday 15:00 - 16:00 or by appointment
**Personal Zoom room:** https://duke.zoom.us/my/k.efstathiou (for online "office hour" meetings)
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**Short bio:** Dr. Efstathiou studied physics all the way up to his PhD, focusing on the more mathematical aspects of physics, especially those related to dynamics and geometry. After his PhD, he has worked in the Mathematics departments of the University of Groningen and the Xi'an Jiaotong Liverpool University. His research interests are mostly in the area of dynamical systems, where concepts from Real Analysis play an essential role. He has taught for several years courses on Metric and Topological Spaces and on Complex Analysis, which present in a different setting many concepts that also appear in Real Analysis. He is looking forward to convey the appreciation for the rigor and abstraction of Real Analysis that are so essential to modern Mathematics.

## What is this course about?

Calculus was developed, starting with Leibniz and Newton, on the foundation of intuitive, and not always very precise or well-defined, concepts. Even basic ideas such as limits and continuity took centuries to be formalized. Major steps in this direction were the introduction of ε-δ definitions by Weierstrass, that did away with the intuitive but imprecise notions of "approaching" a number, or "moving" along the real axis, and the work by Cantor on the properties of real numbers.

Real Analysis can appear at first sight as a more rigorous version of Calculus, where every construction found in Calculus is placed on a firm mathematical basis and hidden assumptions about the nature of the real numbers are revealed. However, Real Analysis is much more than that. The concepts that are introduced to rigorously describe real numbers, subsets of the real line, properties of functions and the convergence of sequences of functions, have powerful generalizations to higher dimensions and to more abstract spaces, that is, in contexts where intuition can (and often does) fail us. Such concepts form the foundation of a large part of modern Mathematics. Especially notions such as compactness, completeness, Cauchy sequences, uniform convergence, etc. are found everywhere in Mathematics. Meeting such concepts for the first time in the familiar context of functions of one real variable helps to build intuition for them before revisiting them in more abstract settings.

Moreover, this course is also an excellent setting for learning how to read and write proofs since we will be able to see several proof techniques that are used throughout Mathematics. Unlike other courses, where either no or few proofs are given, one of the aims in Real Analysis is to prove (almost) everything. Real Analysis is not about developing new computational skills but about developing a mathematical mindset and rigorous thinking that will be applicable for the rest of your careers as applied mathematicians, (data) scientists, engineers, financial analysts, etc.

## What background knowledge do I need before taking this course?

The courses below are the only formal prerequisites of MATH 308.

- ☑ MATH 203 Advanced Calculus or MATH 205 Probability and Statistics (but not MATH 206).

## What will I learn in this course?

After successfully completing the course students will be able to:

1. rigorously formulate logical statements with quantifiers and their negations;
2. define the basic theoretical concepts of Real Analysis that are covered in the lectures (including: limits, continuity, uniform and pointwise convergence, Riemann integrals, completeness, Cauchy convergence, open-closed-compact-perfect-connected sets, series of functions, power series, Taylor series);
3. show whether an object (set, function, sequence, etc.) satisfies a given definition;
4. construct examples of objects (sets, functions, sequences, etc.) satisfying given properties and demonstrating theoretical results, and construct counter-examples showing that a statement is false;
5. reproduce the proofs of basic theoretical results of Real Analysis that are covered in the lectures;
6. apply the theoretical results in specific examples that have not been discussed in lectures and deduce various properties of these examples;
7. prove theoretical results, consequences of the basic theory, that have not been discussed in lectures.

## What will I do in this course?

In each class session I will describe the main relevant ideas for each topic in the course and in some cases delve into more technical aspects of the topic. The purpose of the lectures is to help you get oriented in Real Analysis by presenting the main ideas and how they are related to each other and by working on several problems in the class. You are strongly encouraged to ask questions during class sessions.

The assessment of the learning objectives will be performed through **homework assignments**, two **midterm in-class exams**, and the **final exam**.

All the **exams** will be closed book. Collaboration, finding solutions on the internet or from other sources, and consulting your notes and the textbook are not allowed. It is important to know that the exams will not necessarily contain questions that are copies of (or very similar to) examples discussed in the lectures or are solved in the book, or similar to other exercises you have solved. In the exams you will be asked to apply the concepts (definitions, theorems, other results), that are covered in the studied material to problems that you have not met before but can be solved through the application of these concepts. This is going to be very different than a course such as MATH 105 where, typically, you learn computation techniques and then you are asked to apply them to very similar problems as those you have already discussed.

There will be 7 graded **homework assignments**. At the beginning of each week I will post the homework assignments on Gradescope. Solutions to all the given practice problems must be submitted to Gradescope by Sunday 5pm each week. The day after the deadline I will post complete solutions to the homework problems on Canvas. The homework assignments will be checked for completeness (i.e., that all questions have been answered) and a few (2-3) answers will be checked for correctness. You should also compare your answers with the solutions I will post on Canvas.

## How can I prepare for the class sessions to be successful?

To prepare for the course:

- ☑ Review the syllabus, and make sure you know where to find what material you need to study for each class session, especially for the first one.
- ☑ Review all the material from MATH 101 Calculus and the material from MATH 203 Advanced Calculus or MATH 205 Probability and Statistics related to sequences and series.

To prepare for each class session:

- ☑ Check the syllabus, the reading guide, and Canvas for the material you need to review *before* each class session. Study the indicated textbook sections and watch the related video recordings that can be found on Canvas. Identify the main concepts that will be covered in each session.
- ☑ Review the homework assignment problems **before** class so that you have an idea so that during the class sessions you can make connections between the covered material and solving these problems.
- ☑ Make sure that you have a thorough understanding of the material covered in previous sessions and prepare a list of questions concerning aspects of the material that is still unclear. You can ask these questions during the upcoming session or during office hours.

## What required texts, materials, and equipment will I need?

The (required) main textbook for the course is:

S. Abbott, *Understanding Analysis*, 2nd edition, Springer (2015).
This is a very well written book, providing several discussions, examples, and exercises for the topics that we cover in this course. Moreover, it discusses the context and motivation for the ideas that are presented and explains the role they play in the more general development of Mathematics. The book is available as DRM-free e-book (PDF, ePub).

The following article concerning mathematical writing is also required reading:

F. E. Su, *Some Guidelines for Good Mathematical Writing*, MAA Focus, vol. 35(4), pp. 20-22 (2015).
This is a very brief article covering some of the fundamentals of mathematical writing.

## What optional texts or resources might be helpful?

### *Texts at similar level*

M. C. Reed, *Fundamental Ideas of Analysis*, Wiley (1997).
A great book which, unfortunately, is not available as an e-book. This is the textbook used in the corresponding course (MATH 431) at Duke University.

K. A. Ross, *Elementary Analysis: The Theory of Calculus*, 2nd edition, Springer (2013).
Another very well written book, similar in spirit to the book by Abbott.

### *Advanced texts*

W. Rudin, *Principles of Mathematical Analysis*, 3rd edition (1976).
For many mathematicians this is the classical book from which they have learned Analysis. It is a rather demanding text, often lacking in motivation and in explaining the necessity of certain definitions and approaches. It was instrumental in changing how real analysis was taught when its 1st edition came out in 1953. It is being kept alive through a combination of inertia and nostalgia but it still has its place as a reference book for someone who is already familiar with Real Analysis.

### *Mathematical writing*

The article by Francis Su, mentioned under required texts discusses some of the most fundamental aspects of mathematical writing. Below are some other useful resources on mathematical writing which go into more detail.

F. Vivaldi, *Mathematical Writing*, Springer (2014).
This is a nice resource for learning how to write mathematics "in the small". That is, how to write specific types of mathematical arguments, expressions, sentences, and proofs and it is aimed at undergraduate students who are just beginning to write mathematics. It is available as DRM-free e-book.

K. Houston, *How to Think Like a Mathematician*, Cambridge University Press (2009).
This is not a book about writing *per se*, however, Ch. 2 has useful advice about reading mathematics and Ch. 3 and 4 about writing mathematics. This is an overall very useful book for all mathematics students and I highly recommend it as a general resource.

N. J. Higham, *Handbook of Writing for the Mathematical Sciences*, SIAM, 3rd edition (2020).
This is a comprehensive book about all aspects of mathematical writing. Most relevant for this course are the chapters on the General Principles (Ch. 1), Mathematical Writing (Ch. 2), and maybe English Usage (Ch. 3) and When English is a Foreign Language (Ch. 5). You may also find useful Ch. 8 on TeX, LaTeX, and BibTeX.

S. G. Krantz, *A Primer of Mathematical Writing*, American Mathematical Society, 2nd edition (2017).
This is another comprehensive book on mathematical writing but it puts less emphasis on the fundamentals of writing. Ch. 1, The Basics, is the most useful for this course. Also available on the arXiv as https://arxiv.org/abs/1612.04888 and on this course's Canvas site.

## How will my grade be determined?

The grade will be determined through 5 in-class exams and a final exam.

- ☑ Each of the **two in-class exams** will assess the material covered until that point in the course and each one will contribute **20%** to the final grade.
- ☑ The **final exam** will contribute **35%** to the final grade and it will cover material from the whole course. If the final exam grade is below 50% then the course grade will be F.
- ☑ The **homework assignments** will contribute **25%** to the final grade. Each homework assignment will be graded for completeness (i.e., that all questions have been answered), while a few (2-3) questions will be graded for correctness and for writing.

### *Grading scale*

| Grade | Percentage (p) | Grade | Percentage (p) | Grade | Percentage (p) | Grade | Percentage (p) |
|---|---|---|---|---|---|---|---|
| D+ | 67% ≤ p < 70% | C+ | 77% ≤ p < 80% | B+ | 87% ≤ p < 90% | A+ | 97% ≤ p ≤ 100% |
| D | 63% ≤ p < 67% | C | 73% ≤ p < 77% | B | 83% ≤ p < 87% | A | 93% ≤ p < 97% |
| D- | 60% ≤ p < 63% | C- | 70% ≤ p < 73% | B- | 80% ≤ p < 83% | A- | 90% ≤ p < 93% |
| F | p < 60% or final exam < 50% | | | | | | |

## What are the course policies?

### *Exams*

All exams will be closed book. You may not search online for answers to the exam questions, or give or receive aid for the exam from others, or consult the textbook or any other material. Violations of these policies will lead to a zero for the exam and the Office of Undergraduate Studies will be informed.

### *Assignment deadlines*

All homework assignments should be handed-in by the announced deadline; late submissions will not be graded.

### *Academic Integrity*

As a student, you should abide by the academic honesty standard of the Duke Kunshan University. Its Community Standard states: "Duke Kunshan University is a community comprised of individuals from diverse cultures and backgrounds. We are dedicated to scholarship, leadership, and service and to the principles of honesty, fairness, respect, and accountability. Members of this community commit to reflecting upon and upholding these principles in all academic and non-academic endeavors, and to protecting and promoting a culture of integrity and trust."

### *Academic Policy & Procedures*

You are responsible for knowing and adhering to academic policy and procedures as published in University Bulletin and Student Handbook. Please note, an incident of behavioral infraction or academic dishonesty (cheating on a test, plagiarizing, etc.) will result in immediate action from me, in consultation with university administration (e.g., Dean of Undergraduate Studies, Student Conduct, Academic Advising). Please visit the Undergraduate Studies website for additional guidance related to academic policy and procedures.

*Academic Disruptive Behavior and Community Standard*

Please avoid all forms of disruptive behavior, including but not limited to: verbal or physical threats, repeated obscenities, unreasonable interference with class discussion, making/receiving personal phone calls, text messages or pages during class, excessive tardiness, leaving and entering class frequently without notice of illness or other extenuating circumstances, and persisting in disruptive personal conversations with other class members. Please turn off phones, pagers, etc. during class unless instructed otherwise. Laptop computers may be used only to take notes. If you choose not to adhere to these standards, I will take action in consultation with university administration (e.g., Dean of Undergraduate Studies, Student Conduct, Academic Advising).

*Academic Accommodations*

Duke Kunshan University makes reasonable academic accommodations for qualified students with disabilities. All undergraduate accommodations must be approved through the Student Accommodation Services. Students requesting accommodations for this course should forward their official accommodation letter to the instructor and ask to schedule a time to meet and discuss the implementation of their accommodation(s). It is the student' s responsibility to meet, discuss, and provide an electronic copy of the Instructor Accommodation Letter to each instructor. Accommodations will not be granted retroactively. Accommodations for test, quiz, or exam taking must be arranged with the professor at least a week before the date of the quiz, test or exam, including finals.

## What campus resources can help me during this course?

*Academic Advising and Student Support*

Please consult with me about appropriate course preparation and readiness strategies, as needed. Consult your academic advisors on course performance (i.e., poor grades) and academic decisions (e.g., course changes, incompletes, withdrawals) to ensure you stay on track with degree and graduation requirements. In addition to advisors, staff in the Academic Resource Center can provide recommendations on academic success strategies (e.g., tutoring, coaching, student learning preferences). Please visit the Office of Undergraduate Advising website for additional information related to academic advising and student support services.

*Writing and Language Studio*

For additional help with academic writing—and more generally with language learning—you are welcome to make an appointment with the Writing and Language Studio (WLS). You can register for an account, make an appointment, and learn more about WLS services, policies, and events on the WLS website. You can also find writing and language learning resources on the Writing & Language Studio Sakai site.

*IT Support*

If you are experiencing technical difficulties, please contact IT:

- China-based faculty / staff / students 400-816-7100, (+86) 0512- 3665-7100
- US-based faculty/staff/students (+1) 919-660-1810
- International-based faculty/staff/students can use either telephone option (recommend using tools like Skype calling)
- Live Chat: https://oit.duke.edu/help
- Email: service-desk@dukekunshan.edu.cn

## What is the expected course schedule?

The numbers in the brackets under "class topic / unit name" below refer to sections in the course textbook.

## Week 1

### Class meeting 1-1

| | |
|---|---|
| **Topics** | Course overview: what is Real Analysis?<br>Mathematical Writing<br>Some Preliminaries (1.2)<br>The Axiom of Completeness (1.3) |
| **Pre-class work** | Review syllabus<br>Read "Some Guidelines for Good Mathematical Writing"<br>Complete writing assignment on Canvas<br>Read Sec. 1.1-1.3 from the textbook and watch videos under **Class Meeting 1-1** on Canvas |
| **Planned in-class activities** | Discuss Sec. 1.2, 1.3<br>Work on problems from Sec. 1.2, 1.3 |

### Class meeting 1-2

| | |
|---|---|
| **Topics** | Consequences of Completeness (1.4)<br>Cardinality (1.5)<br>Cantor's Theorem (1.6) |
| **Pre-class work** | Read Sec. 1.4-1.6 from the textbook and watch videos under **Class Meeting 1-2** on Canvas |
| **Planned in-class activities** | Discuss Sec. 1.4-1.6<br>Work on problems from Sec. 1.4-1.6 |

**Deadline** Homework 1 (Sunday 5pm)

## Week 2

### Class meeting 2-1

| | |
|---|---|
| **Topics** | The Limit of a Sequence (2.2)<br>The Algebraic and Order Limit Theorems (2.3)<br>The Monotone Convergence Theorem (2.4, up to Theorem 2.4.2)<br>Subsequences and the Bolzano-Weierstrass Theorem (2.5) |
| **Pre-class work** | Read Sec. 2.1<br>Read the topics from the textbook and watch videos under **Class Meeting 2-1** on Canvas |
| **Planned in-class activities** | Discuss Sec. 2.2-2.5<br>Work on problems related to these topics |

### Class meeting 2-2

| | |
|---|---|
| **Topics** | The Cauchy Criterion (2.6)<br>A First Look at Infinite Series (2.4, starting from Definition 2.4.3)<br>Properties of Infinite Series (2.7) |
| **Pre-class work** | Read the topics from the textbook and watch videos under **Class Meeting 2-2** on Canvas |
| **Planned in-class activities** | Discuss Sec. 2.6, 2.4 (series), 2.7<br>Work on problems related to these topics |

**Deadline** Homework 2 (Sunday 5pm)

## Week 3

### Class meeting 3-1

| | |
|---|---|
| **Topics** | Open and Closed Sets (3.2)<br>Compact Sets (3.3) |
| **Pre-class work** | Read Sec. 3.1<br>Read the topics from the textbook and watch videos under **Class Meeting 3-1** on Canvas |
| **Planned in-class activities** | Discuss Sec. 3.2, 3.3<br>Work on problems related to these topics |

### Class meeting 3-2

| | |
|---|---|
| **Topics** | Perfect Sets and Connected Sets (3.4)<br>Functional Limits (4.2) |
| **Pre-class work** | Read Sec. 4.1<br>Read the topics from the textbook and watch videos under **Class Meeting 3-2** on Canvas |
| **Planned in-class activities** | Discuss Sec. 3.4, 4.2<br>Work on problems related to these topics |

**Deadline** Homework 3 (Sunday 5pm)

## Week 4

### Class meeting 4-1

**Midterm Exam 1** In-class exam, covers material from weeks 1-3

| | |
|---|---|
| **Planned in-class activities** | In-class midterm exam<br>SGIF |

### Class meeting 4-2

| | |
|---|---|
| **Topics** | Continuous Functions (4.3)<br>Continuous Functions on Compact Sets (4.4)<br>The Intermediate Value Theorem (4.5) |
| **Pre-class work** | Read Sec. 4.1<br>Read the topics from the textbook and watch videos under **Class Meeting 4-2** on Canvas |
| **Planned in-class activities** | Discuss the topics<br>Work on problems related to these topics |

**Deadline** Homework 4 (Sunday 5pm)

## Week 5

### Class meeting 5-1

| | |
|---|---|
| **Topics** | Derivatives and the Intermediate Value Property (5.2)<br>The Mean Value Theorems (5.3) |
| **Pre-class work** | Read Sec. 5.1<br>Read the topics from the textbook and watch videos under **Class Meeting 5-1** on Canvas |
| **Planned in-class activities** | Discuss the topics<br>Work on problems related to these topics |

### Class meeting 5-2

| | |
|---|---|
| **Topics** | Uniform Convergence of a Sequence of Functions (6.2)<br>Uniform Convergence and Differentiation (6.3) |
| **Pre-class work** | Read Sec. 6.1<br>Read the topics from the textbook and watch videos under **Class Meeting 5-2** on Canvas |
| **Planned in-class activities** | Discuss the topics<br>Work on problems related to these topics |

**Deadline** Homework 5 (Sunday 5pm)

## Week 6

### Class meeting 6-1

**Midterm Exam 2** In-class exam, covers material from weeks 1-5

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|---|---|
| **Planned in-class activities** | In-class exam |

### Class meeting 6-2

| | |
|---|---|
| **Topics** | Series of Functions (6.4)<br>Power Series (6.5)<br>Taylor Series (6.6) |
| **Pre-class work** | Read the topics from the textbook and watch videos under **Class Meeting 6-2** on Canvas |
| **Planned in-class activities** | Discuss the topics<br>Work on problems related to these topics |

**Deadline** Homework 6 (Sunday 5pm)

**春节 (National Day Holiday)**

## Week 7

### Class meeting 7-1

| | |
|---|---|
| **Topics** | The Definition of the Riemann Integral (7.2)<br>Integrating Functions with Discontinuities (7.3)<br>Properties of the Integral (7.4) |
| **Pre-class work** | Read Sec. 7.1<br>Read the topics from the textbook and watch videos under **Class Meeting 7-1** on Canvas |
| **Planned in-class activities** | Discuss the topics<br>Work on problems related to these topics |

### Class meeting 7-2

| | |
|---|---|
| **Topics** | The Fundamental Theorem of Calculus (7.5)<br>Lebesgue's criterion for Riemann Integrability (7.6) |
| **Pre-class work** | Read the topics from the textbook and watch videos under **Class Meeting 7-2** on Canvas |
| **Planned in-class activities** | Discuss the topics<br>Work on problems related to these topics |

**Deadline** Homework 7 (Sunday 5pm)

## Exam Week

**Final Exam** Wednesday October 18, 15:30 – 18:30, WDR 1007