MATH 409

Topology

Spring 2025, Session 1

Class meetings: Monday and Wednesday 9:30 – 12:00 in WDR 3016.
Academic credit: 4 DKU credits.
Course format: Lecture.

Course Instructor

Instructor: Konstantinos Efstathiou, PhD.
Office: WDR 3107.
Office hours: Monday 12:30 - 14:00 and Thursday 10:00 - 11:30 or by appointment.
Personal Zoom room: https://duke.zoom.us/my/k.efstathiou (for online "office hour" meetings).
E-mail address: k.efstathiou@dukekunshan.edu.cn.
Personal website: https://www.efstathiou.gr/.

Short bio: Dr. Efstathiou studied physics all the way up to his PhD, focusing on the more mathematical aspects of physics, especially those related to dynamics and geometry. His research concerns the topological and geometric aspects of Hamiltonian dynamical systems, and the synchronization of coupled oscillator systems. The topics that will be discussed in this course, point-set topology and algebraic topology, are essential tools in his work on Hamiltonian systems. Before joining DKU, he has worked in the Mathematics departments of Xi'an Jiaotong Liverpool University and the University of Groningen. In Groningen, he taught for several years a course on Metric and Topological Spaces which emphasized point-set topology, and he is excited to be teaching at DKU a similar course which additionally discusses basic concepts from algebraic topology.

What is this course about?

MATH 409 is a first course in topology. Topology is a fundamental area of mathematics that studies continuity, connectivity, compactness, and shape. In addition to being an important field in its own right, topology is used throughout mathematics, in geometry, analysis, algebra, number theory, combinatorics, etc. Topology is also useful in many other scientific fields, like computer science, physics, and, as of late, machine learning. Studying topology develops abstract mathematical thinking and can benefit anyone looking to pursue advanced STEM studies.

This course is divided into several modules. The first module introduces topological concepts by studying metric spaces. The next module dives into abstract topological spaces: what they are, how they can be transformed, and different properties they can have. In the third module, we study the important concepts of compactness and connectedness in topological spaces, and how these relate to products and quotients of spaces. Finally, in the last module, we turn to algebraic topology by studying the fundamental group of a topological space and the elements of homotopy theory. It is this final module that relates to the well-known definition of topology as "stretchy geometry."

Topics include topology of metric spaces, abstract topological spaces, open and closed sets, connectedness, compactness, continuity, completeness, subspaces, product and quotient spaces, separation axioms, homotopies of paths, the fundamental group, covering spaces, index theory, and applications (Borsuk-Ulam Theorem, Ham Sandwich Theorem, Fundamental Theorem of Algebra).

What background knowledge do I need before taking this course?

The formal prerequisites for MATH 409 are:

- [x] MATH 308 Real Analysis, or
- [x] Instructor Consent.

In practice, the most important skill for succeeding in MATH 409 is the ability to understand and write mathematical proofs. Other background knowledge that you need:

- [x] basic knowledge of sets and set-operations, such as subsets, unions, intersections, set differences, complements;
- [x] sequences in $\mathbb{R}$ and convergence of sequences using ε-N arguments; Cauchy sequences in $\mathbb{R}$; subsequences and their convergence properties.
- [x] uniform convergence of sequences of functions $f : \mathbb{R} \to \mathbb{R}$;
- [x] continuity and uniform continuity of functions $f : \mathbb{R} \to \mathbb{R}$ using ε-δ arguments.

What will I learn in this course?

After successfully completing the course, you will be able to:

1. Recall the core definitions and theorems of metric topology and abstract topological spaces.
2. Apply topological techniques and theorems to problems in analysis and algebra.
3. Recall the fundamental definitions of algebraic topology.
4. Compute algebraic topological invariants of simple spaces.
5. Prove several important theorems in topology.

What will I do in this course?

Each week we will have **2 class meetings**. In the class meetings we will review the main definition, theorems, and proofs, and solve problems related to the discussed topics. Several of these problems are important theoretical results in themselves. Moreover, occasionally I will assign specific students to prepare and present a short proof in the following class meeting.

Each week there will be a **homework assignment** — 7 in total. The purpose of the homework assignments is to apply the concepts discussed in the course, think through problems and come up with proofs for different results, and present your proofs clearly and with correct and complete reasoning. You may discuss problems with fellow students but you must submit your own, independently written, proofs. Complete answers to all homework problems will be provided after each homework assignment deadline so you can compare with your answers. There are, however, usually several ways of proving a result. Homework assignments will be graded for mathematical correctness, but also for having a clear logical structure and for the clear presentation of the mathematical arguments.

The course will also have two **midterm exams** and a **final exam**. These will be closed book exams and they will focus mostly on proofs. The two midterm exams will be 90 minutes long, and the final exam will be 180 minutes long. The aim of the exams is to assess your ability to solve problems and to synthesize knowledge in a time-constrained setting and they build upon the in-class discussions and homework assignments.

Finally, you should make ample use of **office hours** and other opportunities to interact and ask questions (e.g., on Canvas discussions). If the given office hours do not work for you, send me an e-mail to arrange an appointment. My role is to guide and support your learning — do not hesitate to come to me with your questions and comments.

How can I prepare for the class sessions to be successful?

To prepare for class sessions:

- ☑ Make sure that you have a thorough understanding of the material covered in previous class meetings and prepare a list of questions concerning aspects of the material that is still unclear. You can ask these questions during the following class meeting or during office hours.
- ☑ Read carefully through the corresponding upcoming sections in the textbook (the lecture schedule is given at the end of the syllabus) and prepare questions to ask during the lectures. I will review this material in every lecture, focusing on the main ideas, but I will not go through every point in detail. If some discussion in the textbook is unclear, you can raise your questions in class (preferable) or during office hours.

To prepare for the midterm and final exams:

- ☑ Review the solved problems from homework assignments and the problems solved in class. Make sure that you have mastered the problems. You can check this by being able to solve the problems without checking at all the given solutions.

What required texts, materials, and equipment will I need?

The following textbook is required:

- **Introduction to Topology** by Theodore W. Gamelin and Robert Everist Greene. 2nd edition. Dover Publications, 1999.

Moreover, it is suggested to:

- Install the Gradescope mobile app.
- Get a (free) Overleaf account.

What optional texts or resources might be helpful?

The following two textbooks may also be useful.

M. A. Armstrong. **Basic Topology**. Springer (1983).
This book focuses more on Algebraic Topology which it develops further than what we discuss in this course, but it covers basic point-set topology very briefly.

Klaus Jänich. **Topology**. Springer (1984).
This book is more typical in the choice of material that it covers, but its approach is more *sui generis* — it is a very interesting book to read.

The following article concerning mathematical writing is also suggested reading:

F. E. Su, **Some Guidelines for Good Mathematical Writing**, MAA Focus, vol. 35(4), pp. 20-22 (2015).
This is a very brief article covering some of the fundamentals of mathematical writing.

How will my grade be determined?

The course grade will be determined through the following graded assessments:

- ☑ Final exam: 35%
- ☑ Midterm exams: 20% × 2
- ☑ Homework assignments: 25% (the lowest of the 7 homework grades will be dropped)

Grading scale

Grade	Percentage (p)	Grade	Percentage (p)	Grade	Percentage (p)	Grade	Percentage (p)
A+	97% ≤ p ≤ 100%	B+	87% ≤ p < 90%	C+	77% ≤ p < 80%	D+	67% ≤ p < 70%
A	93% ≤ p < 97%	B	83% ≤ p < 87%	C	73% ≤ p < 77%	D	63% ≤ p < 67%
A-	90% ≤ p < 93%	B-	80% ≤ p < 83%	C-	70% ≤ p < 73%	D-	60% ≤ p < 63%
						F	p < 60%

What are the course policies?

Communications

If you have a question or discussion topic that you think could be relevant to the whole class then I would like to encourage you to ask it in Canvas Discussions so that everyone can take part. I would also like to encourage you to reply to questions there if you know the answer. For questions that are only relevant to you I prefer e-mail. I usually do not answer e-mails in the evenings and the weekends except for urgent matters.

Assignment deadlines

All homework assignments should be handed-in by the announced deadline; late submissions will not be graded unless an extension has been requested at least 24 hours before the deadline and a convincing explanation has been provided. An extension will never be longer than 24 hours.

It is especially important to submit homework assignments within the given deadline. The reason is that after I have received all assignments I can then start grading and providing feedback and I can also release the solutions to the assignments so that you can compare with your solutions.

Discussion Guidelines

Civility is an essential ingredient for academic discourse. All communications for this course should be conducted constructively, civilly, and respectfully. Differences in beliefs, opinions, and approaches are to be expected. Please bring any communications you believe to be in violation of this class policy to the attention of your instructor. Active interaction with peers and your instructor is essential to success in this course, paying particular attention to the following:

- ☑ Be respectful of others and their opinions, valuing diversity in backgrounds, abilities, and experiences.
- ☑ Challenging the ideas held by others is an integral aspect of critical thinking and the academic process. Please word your responses carefully, and recognize that others are expected to challenge your ideas. A positive atmosphere of healthy debate is encouraged.
- ☑ Read your online discussion posts carefully before submitting them.

Academic Integrity

As a student, you should abide by the academic honesty standard of the Duke Kunshan University. Its Community Standard states: "Duke Kunshan University is a community comprised of individuals from diverse cultures and backgrounds. We are dedicated to scholarship, leadership, and service and to the principles of honesty, fairness, respect, and accountability. Members of this community commit to reflecting upon and upholding these principles in all academic and non-academic endeavors, and to protecting and promoting a culture of integrity and trust."

Academic Policy & Procedures

You are responsible for knowing and adhering to academic policy and procedures as published in University Bulletin and Student Handbook. Please note, an incident of behavioral infraction or academic dishonesty (cheating on a test, plagiarizing, etc.) will result in immediate action from me, in consultation with university administration (e.g., Dean of Undergraduate Studies, Student Conduct, Academic Advising). Please visit the Undergraduate Studies website for additional guidance related to academic policy and procedures.

Academic Disruptive Behavior and Community Standard

Please avoid all forms of disruptive behavior, including but not limited to: verbal or physical threats, repeated obscenities, unreasonable interference with class discussion, making/receiving personal phone calls, text messages or pages during class, excessive tardiness, leaving and entering class frequently without notice of illness or other extenuating circumstances, and persisting in disruptive personal conversations with other class members. Please turn off phones, pagers, etc. during class unless instructed otherwise. Laptop computers may be used only to take notes. If you choose not to adhere to these standards, I will take action in consultation with university administration (e.g., Dean of Undergraduate Studies, Student Conduct, Academic Advising).

Academic Accommodations

If you need to request accommodation for a disability, you need a signed accommodation plan from Campus Health Services, and you need to provide a copy of that plan to me. Visit the Office of Student Affairs website for additional information and instruction related to accommodations.

What campus resources can help me during this course?

Academic Advising and Student Support

Please consult with me about appropriate course preparation and readiness strategies, as needed. Consult your academic advisors on course performance (i.e., poor grades) and academic decisions (e.g., course changes, incompletes, withdrawals) to ensure you stay on track with degree and graduation requirements. In addition to advisors, staff in the Academic Resource Center can provide recommendations on academic success strategies (e.g., tutoring, coaching, student learning preferences). All ARC services will continue to be provided online. Please visit the Office of Undergraduate Advising website for additional information related to academic advising and student support services.

Writing and Language Studio

For additional help with academic writing—and more generally with language learning—you are welcome to make an appointment with the Writing and Language Studio (WLS). You can register for an account, make an appointment, and learn more about WLS services, policies, and events on the WLS website. You can also find writing and language learning resources on the Writing & Language Studio Sakai site.

IT Support

If you are experiencing technical difficulties, please contact IT:

- China-based faculty / staff / students 400-816-7100, (+86) 0512-3665-7100
- US-based faculty / staff / students (+1) 919-660-1810
- International-based faculty / staff / students can use either telephone option (recommend using tools like Skype calling)
- Live Chat: https://oit.duke.edu/help
- Email: service-desk@dukekunshan.edu.cn

What is the expected course schedule?

Numbers of the form I.N below refer to sections in the course textbook. For example, III.2 is the second section of the third chapter.

Week 1

Class meeting 1-1

Topics Course overview. Metric spaces. Open and closed sets.

Pre-class work Review the syllabus. Review section I-1.

Class meeting 1-2

Topics Completeness. Products of metric spaces.

Pre-class work Review convergence and the Cauchy property of sequences of real numbers from MATH 308. Preview sections I-2,4.

Week 2

Deadline Homework 1 (Sunday, January 12, 6pm).

Class meeting 2-1

Topics Compactness. Continuous functions in metric spaces.

Pre-class work Review the ε-δ definition of continuity and the Heine-Borel theorem from MATH 308. Preview sections I-5,6.

Class meeting 2-2

Topics Topological spaces. Base for a topology. Subspaces.

Pre-class work Recall the definitions and properties of open sets in metric spaces from section I-1. Preview sections II-1,2,4.

Week 3

Deadline Homework 2 (Sunday, January 19, 6pm).

Class meeting 3-1

Midterm Exam 1 In-class exam, 90 minutes, covers material from weeks 1-2.

Topics Midterm exam. Continuous functions in topological spaces.

Pre-class work Prepare for the midterm exam. Recall the definition and properties of continuous functions in metric spaces from section I-6. Preview section II-3.

Class meeting 3-2

Topics Separation axioms.

Pre-class work Preview section II-5.

Week 4

Class meeting 4-1

Topics Compactness. Locally compact spaces. Connectedness.

Pre-class work Recall the definition and properties of compact metric spaces. Preview sections II-6,7,8.

Deadline Homework 3 (Sunday, January 26, 6pm).

Spring Festival - 春节

Class meeting 4-2

Topics Path connectedness. Finite product spaces. Quotient spaces.

Pre-class work Recall the definition and properties of connected topological spaces. Preview sections II-9,10,13.

Week 5

Deadline Homework 4 (Sunday, February 9, 6pm).

Class meeting 5-1

Midterm Exam 2 In-class exam, 90 minutes, covers material from weeks 1-4.

Topics Midterm exam. Groups.

Pre-class work Prepare for the exam. If you have taken MATH 401, review the definition and properties of groups. Preview section III-1.

Class meeting 5-2

Topics Homotopic paths. The fundamental group.

Pre-class work Preview sections III-2,3.

Week 6

Deadline Homework 5 (Sunday, February 16, 6pm).

Class meeting 6-1

Topics Induced homomorphisms. Covering spaces.

Pre-class work Preview sections III-4,5.

Class meeting 6-2

Topics Some applications of the index. Homotopic maps.

Pre-class work Preview sections III-6,7.

Week 7

Deadline Homework 6 (Sunday, February 23, 6pm).

Class meeting 7-1

Topics	Maps into the punctured plane. Vector fields.
Pre-class work	Preview sections III-8,9.

Class meeting 7-2

Topics	Vector fields (continued). The Jordan curve theorem.
Pre-class work	Preview sections III-9,10.

Exam Week

Deadline	Homework 7 (Sunday, March 2, 6pm).
Final Exam	Monday, March 3, 15:30 - 18:30, WDR 3016.